

$$X \sim B(n, p) \quad Y = \frac{X}{n}$$

a) $E[Y] = E\left[\frac{X}{n}\right] = \frac{1}{n} E[X] = \frac{1}{n} np = p$

$$\text{VAR}[Y] = \text{VAR}\left[\frac{X}{n}\right] = \frac{1}{n^2} \text{VAR}[X] = \frac{1}{n^2} npq = \frac{pq}{n}$$

* $\text{VAR}[X] = npq$ (VER TAB. DIST. DISCRETAS)

COMO $\text{VAR}[Y^2] = E[Y^2] - (E[Y])^2$ ENTÃO

$$E[Y^2] = \text{VAR}[Y^2] + (E[Y])^2 = \frac{pq}{n} + p^2.$$

↓) SLIDE 124 : $M_{a+bx}(t) = e^{at} M_x(bt)$

COMO $M_X(t) = (q + p e^t)^n$

ENTÃO $M_{\frac{X}{n}}(t) = e^0 (q + p e^{\frac{1}{n}t})^n$

OU SEJA $M_Y(t) = (q + p e^{\frac{t}{n}})^n$